

# BUBBLE GROWTH AND HEAT-TRANSFER MECHANISMS IN THE FORCED CONVECTION BOILING OF WATER CONTAINING A SURFACE ACTIVE AGENT

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**Abstract**—An experimental investigation of forced convection boiling in a vertical annulus is reported. The boiling fluid was water both pure and with various amounts of surface active agent added to reduce surface tension. In addition to surface tension, the velocity and subcooling were also parametrically varied.

At the same wall temperature, a higher heat flux and a larger bubble population were measured in boiling of water having reduced surface tension. Also the bubble growth rate was slower than that in pure water, by at least a factor of ten, and the bubbles formed in unison rather than at individual sites.

The difference in this bubble growth from that of pure water was attributed to different energy transfer processes controlling the bubble growth. It is hypothesized that heat conduction rate in the surface controls the bubble growth in pure water and the evaporation rate in water containing an additive. A model assuming this hypothesis is proposed and an order of magnitude analysis is given for verification of the model.

It is concluded that latent energy transport contributes approximately 50 per cent of the measured heat flux in water without a surface active additive and significantly more in water containing an additive.

## NOMENCLATURE

<p><math>A_b</math>, surface area of a bubble [<math>\text{ft}^2</math>];</p> <p><math>\bar{A}_b</math>, time average surface area of a bubble [<math>\text{ft}^2</math>];</p> <p><math>C_L</math>, specific heat of the liquid [<math>\text{Btu/lb degF}</math>];</p> <p><math>D_b</math>, diameter of a bubble [<math>\text{ft}</math>];</p> <p><math>h_{fg}</math>, latent heat of vaporization [<math>\text{Btu/lb}</math>];</p> <p><math>K</math>, thermal conductivity [<math>\text{Btu/h-ft}^2</math>];</p> <p><math>K</math>, Boltzmann's constant;</p> <p><math>M</math>, mass transferred through the bubble [<math>\text{lb}</math>];</p> <p><math>m</math>, mass of a molecule [<math>\text{lb}</math>];</p> <p><math>n</math>, number of molecules per unit volume [<math>1/\text{ft}^3</math>];</p> <p><math>q/A</math>, heat flux [<math>\text{Btu/h-ft}^2</math>];</p>	<p><math>(q/A)_C</math>, convective heat flux [<math>\text{Btu/h-ft}^2</math>];</p> <p><math>(q/A)_L</math>, latent heat flux [<math>\text{Btu/h-ft}^2</math>];</p> <p><math>t</math>, time [<math>\text{s}</math>];</p> <p><math>t_w</math>, waiting period for bubble growth [<math>\text{s}</math>];</p> <p><math>T</math>, temperature [<math>^{\circ}\text{F}</math> or <math>^{\circ}\text{R}</math>];</p> <p><math>T_B</math>, temperature of bulk liquid [<math>^{\circ}\text{F}</math> or <math>^{\circ}\text{R}</math>];</p> <p><math>T_{sat}</math>, saturation temperature [<math>^{\circ}\text{F}</math> or <math>^{\circ}\text{R}</math>];</p> <p><math>T_w</math>, wall temperature [<math>^{\circ}\text{F}</math> or <math>^{\circ}\text{R}</math>];</p> <p><math>T'_w</math>, wall temperature after the initial bubble growth period [<math>^{\circ}\text{F}</math> or <math>^{\circ}\text{R}</math>];</p> <p><math>V_b</math>, volume of a bubble [<math>\text{ft}^3</math>];</p> <p><math>\bar{v}</math>, average molecular velocity [<math>\text{ft/s}</math>];</p> <p><math>x</math>, distance measured perpendicularly from the solid heating surface or bubble interface [<math>\text{ft}</math>].</p>
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## Greek symbols

$\alpha$ ,	evaporation coefficient;
$\delta$ ,	thickness of superheat liquid layer [ $\text{ft}$ ];
$\theta$ ,	temperature difference ( $T - T_B$ ) [ $\text{degF}$ or $\text{degR}$ ];

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$\theta_s$ ,	temperature difference ( $T_{\text{sat}} - T_B$ ) [degF or degR];
$\theta_w$ ,	temperature difference ( $T_w - T_B$ ) [degF or degR];
$\lambda$ ,	thermal diffusivity [ft <sup>2</sup> /h];
$\rho_L$ ,	density of the liquid [lb/ft <sup>3</sup> ];
$\rho_v$ ,	density of the vapor [lb/ft <sup>3</sup> ];
$\sigma$ ,	liquid-vapor surface tension [dyn/cm];
$\tau$ ,	period of bubble growth or bubble cycle [s].

### INTRODUCTION

CONSIDERABLE disagreement exists in the current literature regarding the fraction of the total nucleate boiling heat flux that can be attributed to latent energy transport. Rohsenow [1], Forster and Greif [2], and others have concluded that latent energy transport is negligible in comparison to that transferred by bubble induced convection. Equations predicting the rate of bubble growth and related boiling phenomena have been derived on the basis of heat conduction theory. Zuber [3] equated the transient heat conduction from the liquid at the bubble interface to the latent energy increase of the bubble plus the heat transfer to the bulk liquid and derived an equation which predicts the growth of bubbles in saturated pool boiling quite well. Hsu and Graham [4] modified Zuber's work by analyzing the liquid layer in contact with the bubble interface as a transient flat plate heat conduction problem, and thus, were able to calculate the heat-transfer rate to the bulk liquid. Also they included in their bubble growth analysis a term representing heat transfer through the base of the bubble which had been ignored by Zuber.

Han and Griffith [5], following the conduction approach, calculated the nucleate boiling heat flux as the energy carried away from the surface by the growth of each bubble

$$Q = \int_0^{\infty} (T - T_B) C_L \rho_L dx,$$

where the temperature distribution ( $T - T_B$ ) is

that at a bubble site just prior to growth, which comes from solution of the transient conduction equation (semi-infinite liquid slab) subject to the boundary condition of constant wall temperature and the initial condition that at bubble detachment liquid at uniform bulk temperature flows in and contacts heated surface. Calculated total boiling heat flux based on measured bubble sizes, frequencies and populations agreed well with measured flux.

The above cited reference does not include, in their heat-transfer analyses, latent energy transport by simultaneous evaporation near the base of the bubble and condensation at the top which Bankoff [6] suggests as a possible mechanism of heat transfer that can occur in conjunction with bubble growth. Such a concept is also supported by the measurements of severe local heater surface temperature decreases beneath growing vapor bubbles reported by Moore and Mesler [7] and Hendricks and Sharp [8]. These authors concluded that the observed surface temperature drops were due to large quantities of latent energy, required to sustain bubble growth, being drawn from the heated surface over a very short period of time and thus implying that a major portion of the total energy is transferred through the interior of the bubble rather than through the liquid. Thus diverse theories exist.

The present work, aimed at studying the effect of reduced liquid-vapor surface tension on forced convection boiling of water, has produced some interesting data which may help to resolve this disagreement. High-speed cinematography showed that in water containing a surface active agent (to reduce surface tension) the bubble formation was considerably different from that in pure water, and in particular growth and collapse rates were less by an order of magnitude. The presence of large surface active-agent molecules in a liquid-vapor interface will greatly restrict the evaporation rate [9], thus it was hypothesized that the energy transfer mechanism controlling the rate of bubble growth for water containing an additive

was evaporation; whereas for pure water, the bubble growth would be controlled by the rate at which energy can be transferred by conduction to the liquid-vapor interface. Based on this hypothesis a model describing the mechanism of heat transfer occurring in pure distilled water and water containing an additive is suggested. This model attributes significant contributions to the total heat flux both by energy transport through the bubble and by bulk liquid convection due to bubble-produced turbulence. The energy transfer requirements of the model are related to the experimentally measured surface temperature drops beneath growing bubbles as reported in the literature.

First, however, the experimental arrangement is described and the results upon which the model is based are cited.

#### EXPERIMENTAL ARRANGEMENTS

The flow loop (1-in O.D. brass tubing) is shown schematically in Fig. 1. In a vertical annular test section (Fig. 2) the following ranges of variables were obtained: heat fluxes up to  $2 \times 10^6$  Btu/h ft<sup>2</sup>; inlet temperature 100–170°F; average fluid velocity 0.8 to 4 ft/s; pressure 30 psia.

Flow supplied by a 15 hp, 500 g/m centrifugal

pump, was measured with a sharp edged orifice, calibrated in place. To reduce flow oscillations and flow excursions in the test section, a major pressure drop (75 psia) was maintained across a throttling valve located directly preceding the test section. Test section inlet temperature was controlled to  $\pm 2$  degF by temperature controller which actuated cooling water flow through a heat exchanger in the surge tank reservoir.

The annular test section (Fig. 2) was jacketed with a transparent 1.5-in I.D. by 30 in Pyrex glass tube to permit visual and photographic study. The inner (heated) surface was made from  $\frac{3}{16}$ -in O.D., 16 mil wall, stainless type 304 and carried direct heating current supplied by a motor-generator (1500 A max.). Brass tubes, 7-in long with a 0.030-in wall thickness, were silver soldered to the ends of the stainless steel tube, thus separating it from the entrance and exit regions of the test section. The heater was positioned in the test section through "O" ring sealed slip joints which allowed for thermal expansion and ease of assembly. The electrical contact of the slip joints was periodically checked by measuring the temperature profile across the slip joint with a movable thermocouple probe. These tests were carried out with the system operating at various heat flux and

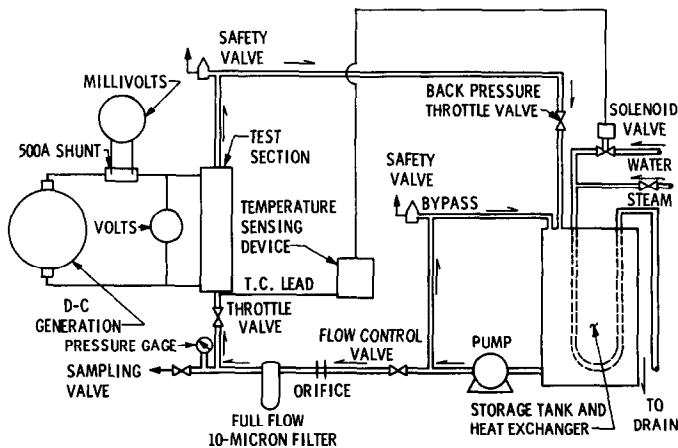


FIG. 1. Schematic diagram of flow loop.

no overheating of the joint was ever experienced.

### Photography

A Hycam 16-mm Model K1004 high-speed

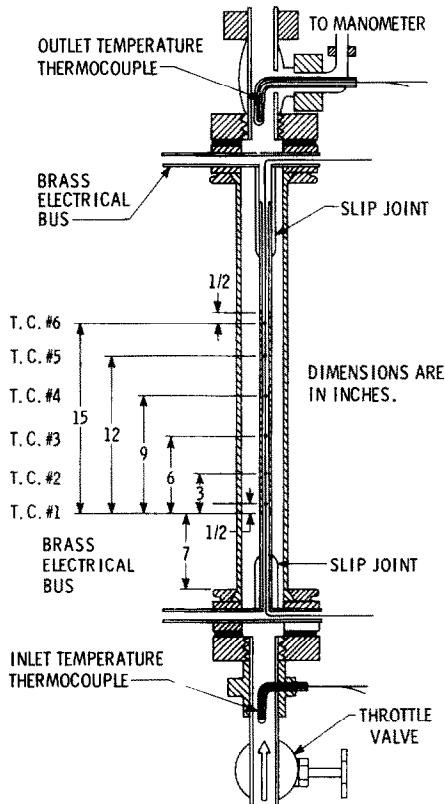


FIG. 2. Test sections. (Dimensions in inches.)

motion picture camera gave a maximum of 7500 pps. Millisecond timing marks permitted measurements of camera speeds to  $\pm 0.1$  per cent. The area of view (1.0 in by 0.7 in) was centered on the heater approximately 9 in from the test section exit and illuminated by four GEBFJ (750 W) photo lamps.

Projection of the developed motion picture film employed a 16-mm Model 173 Bell and Howell "Time and Motion Study Projector" which was equipped with a frame counter and with a hand crank for individual frame projection. Bubble growth cycles were conveniently obtained by recording the number of frames the

bubble remained on the screen and computing the physical time from the timing marks on the film.

Bubble dimensions were scaled from a Craig 16-mm Model V-46 film viewer with a 4 in by 6 in viewing screen. The true dimensions were then computed by multiplying the scaled value by the ratio of actual heater diameter to heater diameter as scaled from the viewer. Standard methods of error analysis indicate a maximum  $\pm 5$  per cent error is to be expected by this method. The bubbles measured were randomly selected by turning the viewer arbitrarily to some frame and selecting a bubble from various locations on the heater surface.

Bubble populations were obtained by placing a transparent square of known dimensions on the viewing screen and counting the number of bubbles appearing in the square per frame. The bubbles per frame were plotted and the time average number of bubbles determined by planimetry of the area under the curve.

### Instrumentation

Inside heater wall temperatures were measured with six iron-constantan thermocouples located inside the stainless steel heater. Corrections for the temperature drop through the heater wall were numerically computed using variable thermal and electrical conductivity. Stainless steel tube well enclosed iron-constantan thermocouples were used to measure inlet and exit bulk water temperatures.

### Experimental procedure

Seventy-six runs were made varying the parameters liquid velocity, inlet temperature and liquid-vapor surface tension. Runs were made at a constant test section pressure of 30 psia for velocities of 0.8, 2.0 and 4.0 ft/s for inlet temperatures of 100, 135, and 170°F and surface tension ranging from 72 to 30 dyn/cm.

During any individual run the heat flux was varied from initially nonboiling conditions to burnout, the above parameters being constant.

Each heater was stabilized by initially in-

creasing the heat flux to the vigorous boiling region, to drive off the inert gases trapped on the heater surface, and then reducing the heat flux to the forced convection region to begin each run.

The average of surface tension measurements on five samples of the liquid taken periodically during a run referenced to room temperature was recorded. A thorough discussion as to the value of surface tension measured (e.g. whether static or dynamic) and the significance of the reference temperature is given in [10]. The significance of the reduced surface tension lies in the fact that a surface active agent ("Ultra Wet 60L" was used here) achieves this result by the migration of long-chain molecules to the liquid-vapor interface.

RESULTS

Plots of heat flux vs.  $\Delta T_{sat}$  shown in Figs. 3 and 4, are representative of the data. Values plotted are local values based on the temperature

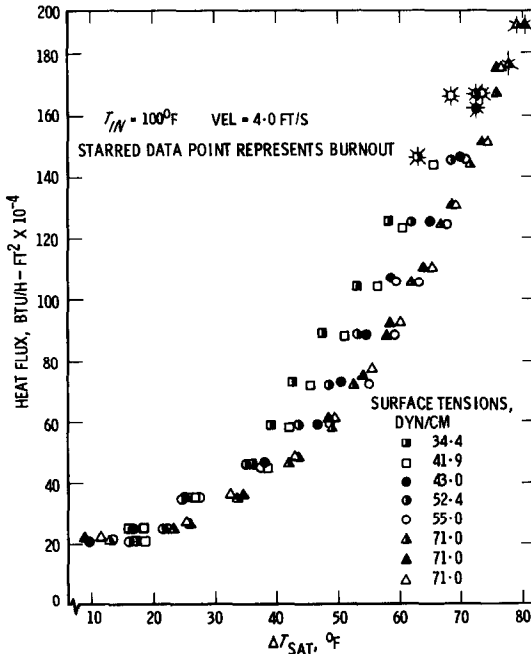


FIG. 3. Forced convection boiling of water having reduced values of surface tension due to the addition of a surface active agent.

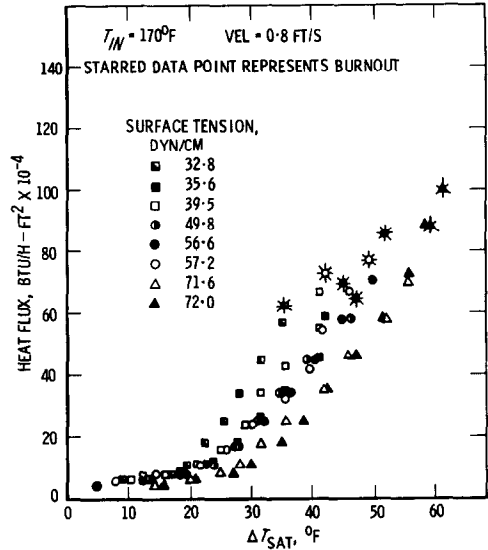


FIG. 4. Forced convection boiling of water having reduced values of surface tension due to the addition of a surface active agent.

readings of either thermocouples number 3 or number 4. Both figures show that at the same wall temperature, a higher nucleate boiling heat flux was obtained for runs made with water having lower values of surface tension.

Figure 5 shows the effect of bulk liquid temperature on the heat flux vs.  $\Delta T_{sat}$  curve for pure water runs and for runs where the surface

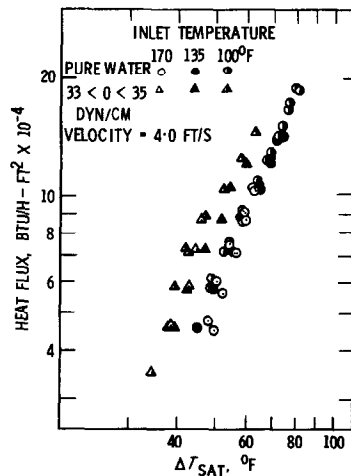


FIG. 5. Illustrates the insensitivity of the boiling curve to inlet temperature.

tension was reduced to approximately 34 dyn/cm. Variations in surface tension are observed to shift the boiling curve, but temperature variations at a given value of surface tension have no apparent effect. This suggests that surface tension plays a role in the heat-transfer process associated with boiling only in the superheated thermal layer adjacent to the heated surface.

The ebullition process associated with the higher heat flux (for reduced surface tension) was characterized by very large bubble populations. In fact, when the surface tension was reduced below approximately 60 dyn/cm, the heated surface became completely covered with bubbles tightly packed on the surface. No significant coalescence of the bubbles occurred. The dimensions of the bubbles at lower surface tension did not change appreciably from the dimensions of those witnessed in the pure water test (cf. Figs. 6 and 7).

A particularly interesting phenomenon observed was that the total bubble population grew and collapsed cyclically in all runs. A sequence of photographs from the high-speed motion pictures, which illustrate this cyclic growth, is shown in Figs. 6 and 7. It is noteworthy that a basic difference was observed for pure water and for water having reduced surface tension. In pure water the ebullition process was periodically suppressed, causing the heated surface to become completely bare or free of bubbles. The frequency of suppression decreased with heat flux as shown in Fig. 8. The growth rates for individual bubbles which grew and collapsed at random between periods of suppression, is shown in Fig. 9.

For reduced surface tension the frequency increased with heat flux as is shown in Fig. 10. Moreover, the bubbles did not grow and collapse individually but all grew in unison and remained on the surface for the duration of the cycle. The bubble population was so large that from approximately 2 msec after bubble growth began, the heated surface was smothered with bubbles, permitting very little liquid to contact

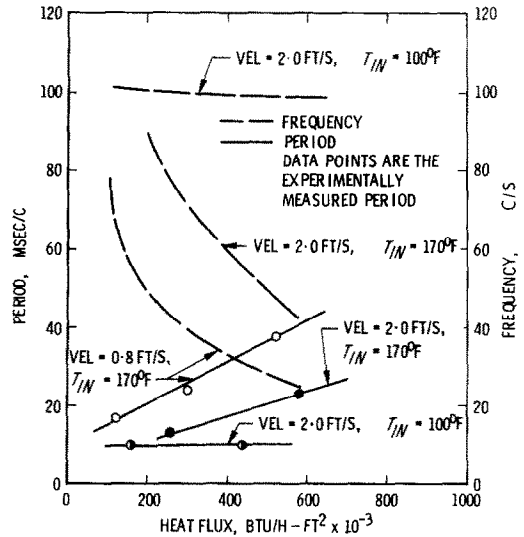


FIG. 8. Period of bubble generation cycle in pure water ( $\sigma = 72$  dyn/cm).

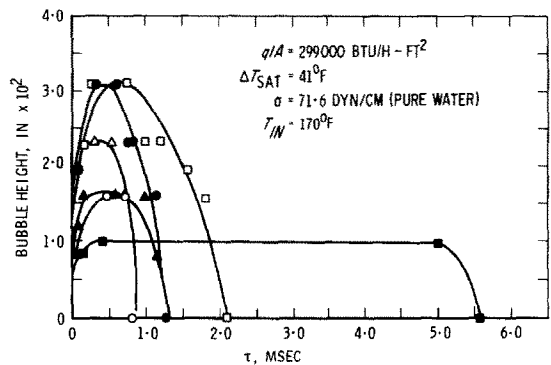


FIG. 9. Individual bubble growth rates (pure water).

the heater until all bubbles collapsed leaving the heated surface again entirely bare. Figure 11 illustrates the rate at which the height of the bubbles varied during some typical cycles.

The gross bubble cycle was observed up to burnout conditions in water containing an additive. However, in pure water it disappeared at a heat flux near  $6 \times 10^5$  Btu/hft<sup>2</sup>, coinciding with a change in the formation of the larger vapor masses which occurred at higher heat fluxes.

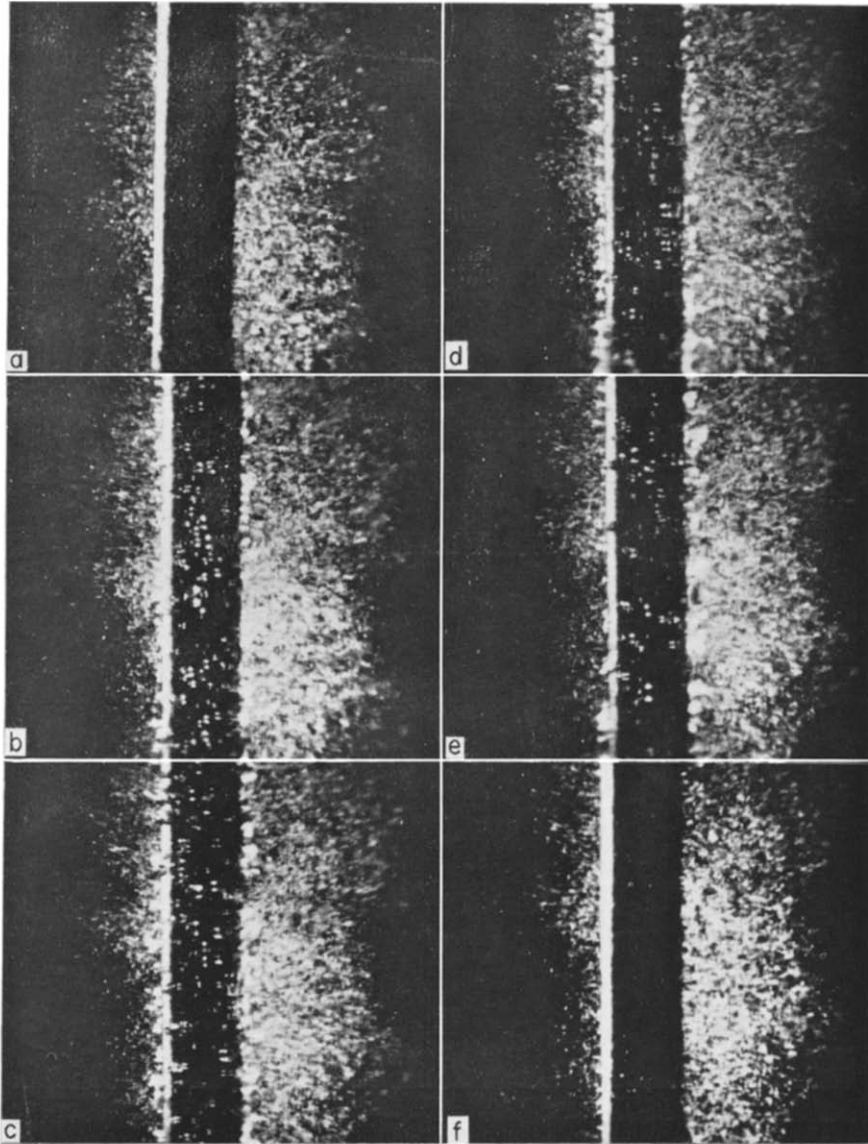


FIG. 6. Cyclic bubble growth in pure water.

(a)  $t = 0.000s$

(b)  $t = 0.045s$

(c)  $t = 0.089s$

(d)  $t = 0.0134s$

(e)  $t = 0.0178s$

(f)  $t = 0.0222s$

Conditions:

Vel = 0.8 ft/s

$q/A = 299\ 100$  Btu/h ft<sup>2</sup>

$T_{in} = 170^{\circ}F$

$\sigma = 71.4$  dyn/cm

$\Delta T_{sat} = 41$  degF.

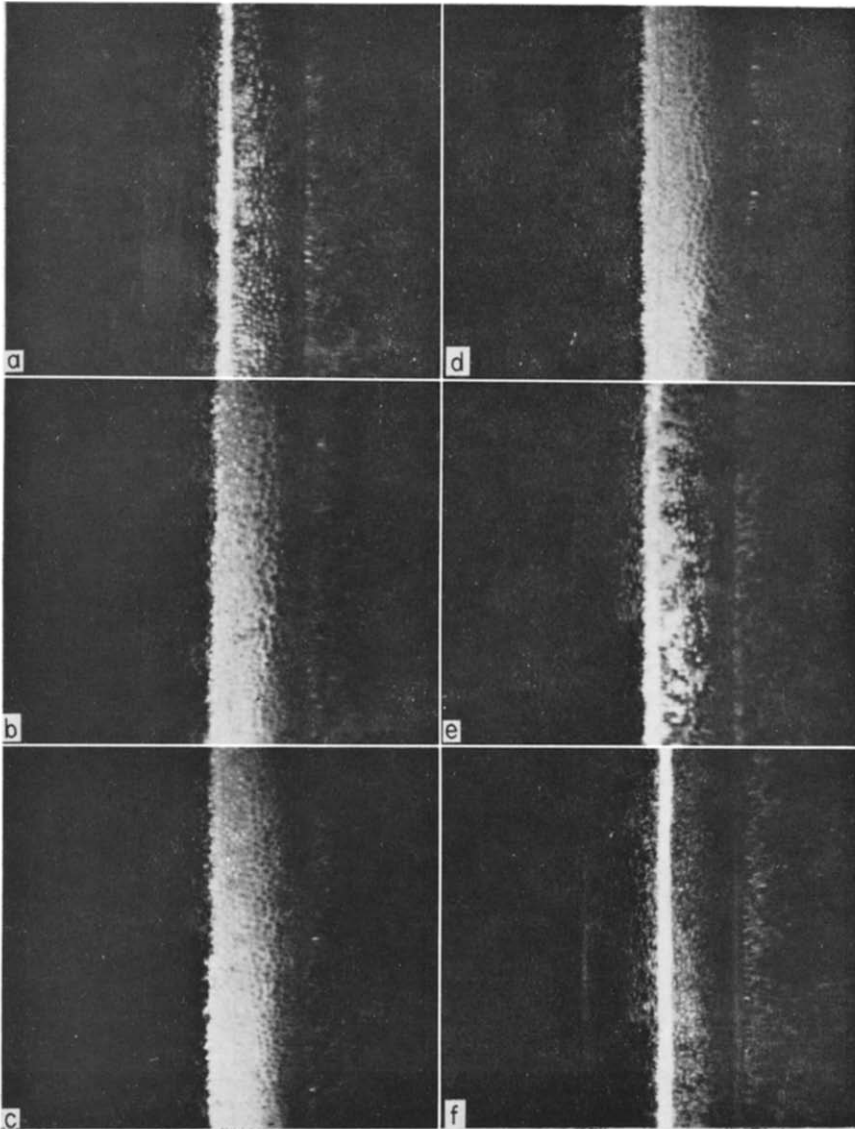


FIG. 7. Cyclic bubble growth in water with reduced surface tension.

(a)  $t = 0.000s$

(b)  $t = 0.007s$

(c)  $t = 0.014s$

(d)  $t = 0.029s$

(e)  $t = 0.036s$

(f)  $t = 0.043s$

Conditions:

Vel = 0.8 ft/s

$q/A = 112\,300$  Btu/h ft<sup>2</sup>

$T_{in} = 170^{\circ}\text{F}$

$\sigma = 34.4$  dyn/cm

$\Delta T_{sat} = 17$  degF.



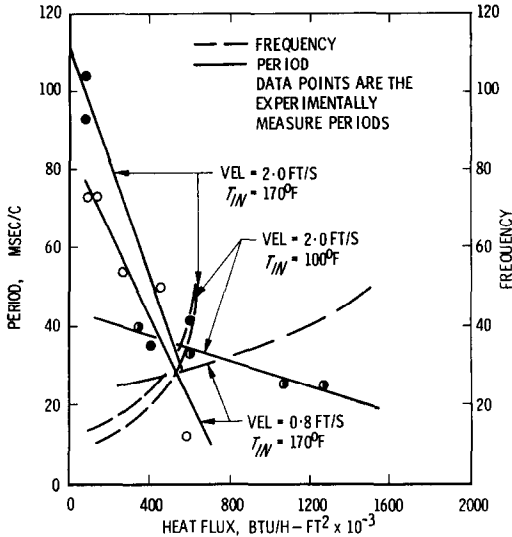


FIG. 10. Period of bubble generation cycle at low values of surface tension ( $\sigma = 34.4$  dyn/cm).

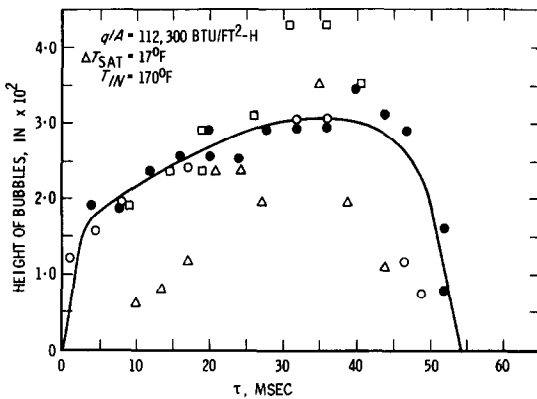


FIG. 11. Bubble growth rate ( $\sigma = 34.4$  dyn/cm).

DISCUSSION OF RESULTS

When the methods of calculating bubble growth rates proposed in [4, 5] are applied to the data reported here, good agreement between the calculated and measured results is obtained only for the pure water tests. For conditions where surface tension is substantially reduced the proposed methods of calculation fail, apparently because the latent energy transport through the bubble is neglected. The high-speed

motion pictures reveal that at low values of surface tension the heated surface becomes completely covered with vapor bubbles between which, at most, only small quantities of trapped liquid contact the heated surface. Thus energy transfer by direct conduction through the liquid is insufficient to account for the experimentally measured heat flux. Moreover, due to the very short delay period in the cycle before bubble growth (approximately 2 msec), it is also impossible for the total energy to be stored in the liquid and then pumped from the surface by the growing bubbles.

These observations suggest that a different mechanism (conduction) controls the rate of bubble growth and associated heat flux in nucleate boiling of pure water, than that (evaporation) which controls in nucleate boiling of water containing a surface active agent. This can best be explained by the following argument. Consider a bubble growing on a heated surface. The base of the bubble near the heater is surrounded by liquid layer at a temperature higher than the saturation temperature ( $T_{sat}$ ) and the top of the bubble is, in a subcooled system, at a temperature less than  $T_{sat}$ . Under these conditions evaporation occurs at the interface at the base of the bubble whereas condensation occurs at the top interface. Energy is thus carried through the bubble by mass transport. If the rate of evaporation and condensation is high, energy may be transferred at an extremely high rate. The result is that the conduction of heat to the base of the bubble, either from the heated surface or from the surrounding superheated liquid, is not fast enough to keep pace. Consequently the surroundings at the base of the bubble are cooled and the entire bubble interface eventually reaches a temperature where the bubble completely condenses or collapses.

Next, consider that the bubble interface contains foreign molecules such as are present in a fluid containing a surface active agent. These foreign molecules are known to greatly restrict the evaporation rate at the interface

with the result that the energy transfer through the bubble becomes relatively slow. Under these conditions, heat can be conducted to the base of the bubble interface as fast as it is carried away by latent transport through the bubble. Hence the base of the bubble will not be cooled below  $T_{\text{sat}}$  and it will not collapse unless the liquid supply at the base is exhausted.

This example depicts how the bubble growth and the rate of heat transfer can be controlled either by the rate of conduction or by the rate of evaporation. Employing this concept, a model of the boiling process is proposed which explains the different bubble configurations shown in Figs. 6 and 7 and the different bubble growth rates shown in Figs. 9 and 11. Also a qualitative description of the cyclic generation of the gross bubble population, for which the period is recorded in Figs. 8 and 10, is given.

#### DESCRIPTION OF PROPOSED MODEL

*Pure water.* The pure water model follows the general line of reasoning presented in the current literature. The order of magnitude values quoted throughout the following discussion are calculated in Appendix A and are for a run made with pure water at a measured heat flux of 229 000 Btu/hft<sup>2</sup>, a measured wall temperature of 290°F, and a liquid inlet temperature of 170°F. The data for this run are given in Figs. 6 and 9.

The waiting period between bubbles, during which energy is transferred to the liquid by conduction, is depicted in Fig. 12(a). When the temperature distribution is correct a bubble is nucleated and at first grows very rapidly (Fig. 12b) because it is completely surrounded by superheated liquid (energy is transferred to the bubble interface from all directions). This corresponds to the initial steep portion of the bubble growth curves (where the bubble grows to approximately 80 per cent of its total height in 0.1 msec., see Fig. 9). The heat flux required to support such a growth rate is calculated to be approximately 570 Btu/sft<sup>2</sup> ( $20 \times 10^6$  Btu/hft<sup>2</sup>, see Appendix A). With such a high local heat flux,

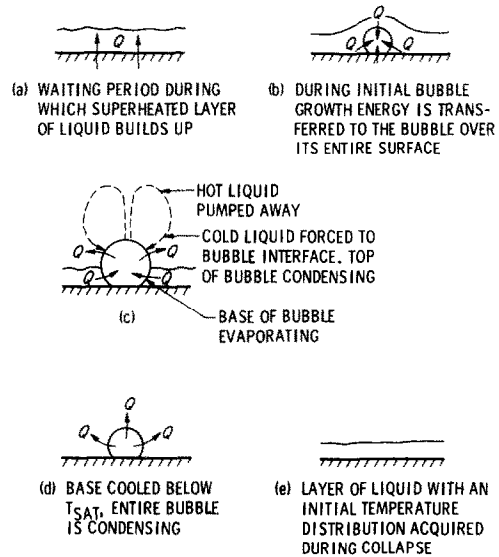


FIG. 12. Model of bubble growth in pure water.

the heater surface temperature will drop, locally, on the order of approximately 15 degF. A temperature drop of the same magnitude takes place in the liquid surrounding the bubble. When this temperature drop occurs in the medium surrounding the bubble, the bubble growth rate decreases appreciably. However, the liquid above the bubble has considerable momentum (the liquid at the bubble interface would have an average velocity equal to the ratio of maximum bubble radius to the growth time; approximately, in this case, 7 ft/s) and is forced into the colder fluid stream causing liquid, at essentially  $T_B$ , to circulate back to the bubble interface. Thus energy stored in the liquid during the waiting period is carried away from the surface by bulk convection. This will be discussed subsequently.

The important result to be noted here is that the top of the bubble is now at a temperature  $T_B$  while the base is at a temperature greater than  $T_{\text{sat}}$ . Consequently, one can postulate that energy will continuously be transported through the bubble, at some rate, until the entire bubble interface is below saturation

temperature, at which time the bubble will have condensed. An approximate analysis (Appendix A) shows that this heat transfer is on the order of 38 Btu/sft<sup>2</sup> ( $1.4 \times 10^6$  Btu/hft<sup>2</sup>). Although this value is considerably less than the initial 570 Btu/sft<sup>2</sup>, it is still sufficient to further cool the surface another 8 degF during the remainder of the bubble lifetime (on an average of 1.87 msec). The energy generated in the heater during the period of bubble growth is small compared to that required to produce the bubble. Therefore, the energy for bubble growth and for evaporation through the bubble must come from the thermal capacity of a very thin surface layer of the heater. This implies that bubble generation is dependent on the rate at which the energy of the surface layer is diminished and replenished. Later, on the basis of this fluctuation in surface energy, a qualitative description of the gross bubble cycle observed in conjunction with boiling of pure water will be given.

Comparison of the calculated surface temperature drops described above to the results reported by Moore and Mesler [7] is interesting. Their sketch of an oscilloscope trace of heater surface temperature measured by a fast response thermocouple is given in Fig. 13. One observes a sharp initial decrease which can be considered as corresponding to the initial rapid 15 degF temperature drop, calculated to occur in 0.1 msec, in this analysis. The decreased rate of change following the large initial drop corresponds in turn to the 8 degF drop calculated to take place during the remainder of bubble life period. One also notes that the magnitude of

the calculated temperatures agree well with those in Fig. 13.

The total latent heat transfer can now be determined by combining the heat flux for both portions of the bubble life and multiplying the result by the time averaged number of bubbles. This accounts for approximately 50 per cent of the total measured heat flux. Returning now to the calculation, in Appendix A, of the heat carried away by convection using essentially the approach of Han and Griffith [5], appropriately modified for subcooling, we find that this amounts to approximately the additional 50 per cent.

Of course, severe assumptions are involved in the approximate analysis used to determine the magnitude of the latent heat transport. However, the results do suggest that both latent heat transport and bulk convection contribute significantly to the total heat flux and lead to a plausible explanation of the high heat flux experimentally measured in this work. It is realized that in other experiments reported in the literature the measured heat flux is accounted for by bulk convection alone. We believe this can be explained as follows. Generally, results reported in the literature pertain to the saturated boiling where  $T_B = T_{sat}$ . Examination of equation (6), Appendix A, shows that energy transport through the bubble can be considered, at least for an order of magnitude analysis, dependent on the factor  $[(\sqrt{T_w}) - (\sqrt{T_B})]$  where the temperatures are absolute. With  $T_B = T_{sat}$  this factor is quite small, hence in saturated boiling studies the latent energy transport can be neglected without appreciable error. Thus it would appear that the heat flux is a relatively strong function of  $T_B$  which is not confirmed by experiment. However, other factors such as bubble size, bubble growth rate, decay period, number of bubbles and waiting period all vary with  $T_B$ . In particular, variation in bubble size and decay period can balance variations in  $[(\sqrt{T_w}) - (\sqrt{T_B})]$ . Consequently, the proposed model is not contradictory to experimental findings as it might seem.

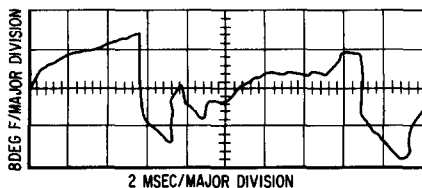


FIG. 13. Typical photographs of the oscilloscope face showing the surface temperature behavior [4].

*Case of reduced surface tension.* Data for bubbles growing in water having reduced surface tension due to the addition of a surface active agent are given in Figs. 7 and 11. The bubble configuration is different from that associated with nucleate boiling in pure water (Figs. 6 and 9). This is believed to be due to the reduction of evaporation rate produced by the surface active agent long-chain molecules, present in the liquid-vapor interface. Davies and Rideal [9] state that such molecules can increase the resistance to evaporation by as much as a factor of  $10^4$ . Consequently, the growth of a bubble due to evaporation in water containing an additive can be appreciably decreased as was experienced in the present investigation. Moreover, when the rate of evaporation is slow compared to the rate at which energy can be transferred by conduction through the fluid, or can be generated in the heater, the growth of a bubble will not markedly lower the temperature of its surroundings as was shown to be the case for bubble growth in pure water. Hence when a slow growing bubble of this type is in a region where the top of the bubble is condensing and the base is evaporating, the bubble may remain essentially in a state of equilibrium as long as liquid is available at the base to replace that lost to evaporation.

To account for the above considerations, the bubble model proposed in the section dealing with pure water is modified as depicted in Fig. 14. Numerical values quoted in the discussion are taken from Figs. 7 and 11, which are typical. In Fig. 14(a), as in the pure water case, a superheated liquid layer is required before a bubble will grow. However, as reduced surface tension improves nucleation, the superheat required is smaller than that required in pure water. Hence the waiting period becomes very short (measured as approximately 2 msec) and the amount of energy transferred to the liquid by conduction is small. In turn, the energy transferred by pumping of the bubbles becomes small. Use of equation (11), Appendix A, shows pumping

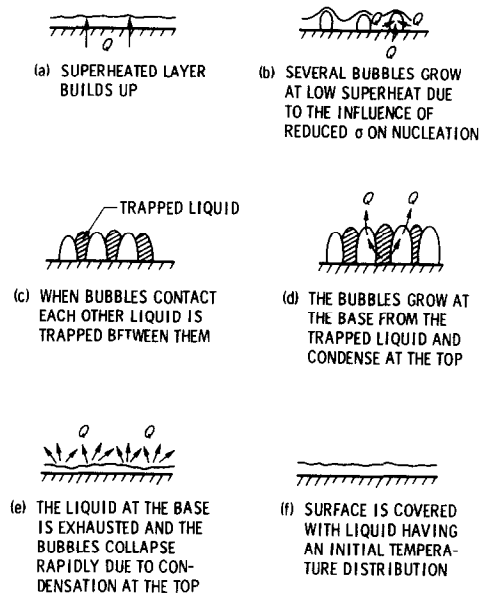


FIG. 14. Model of bubble growth in water with reduced surface tension.

would contribute approximately 20 per cent of the measured heat flux only (measured heat flux was  $112000 \text{ Btu/hft}^2$ ).

All the bubbles grow in unison as shown in Fig. 14(b). This is different from pure water where bubbles grew randomly from individual active sites and is caused by the reduced surface tension creating a multiplicity of nucleation sites. Figure 11 shows, as was the case with pure water, that the initial bubble growth is very fast. Again, the reason for this is that the bubble is growing from energy transferred to it over the entire interface. The bubble grows to approximately 50 per cent of maximum height in 3 msec (Fig. 11). Note this growth period, although short relative to the total bubble life period, is thirty times longer than the initial growth period measured for pure water. Although it must be pointed out that this comparison is made for different values of heat flux, the extreme difference in growth time is believed to result mainly from the restricted evaporation rate. Because the number of bubbles forming in unison is large, they interfere or

contact one another within approximately 3 msec from the start of growth. It is postulated that when this interface occurs liquid is trapped between the bubbles; and at nearly the same time, the top of the bubble breaks through the superheated liquid layer and begins to condense.

The liquid trapped at the base evaporates and diffuses through the bubble carrying with it energy. Since the evaporation rate is slow, the energy drawn from the neighborhood of the bubble base is replenished by conduction, and the temperature at the base remains almost constant. Calculations in Appendix B show that the initial bubble growth, before the top begins to condense, results in a surface temperature drop of approximately 5 degF. However, following this only a slight drop occurs for the remainder of the bubble life. Hence, the base of the bubble is continuously above  $T_{\text{sat}}$  and the bubble is in a state of near equilibrium, growing only slightly as shown by Fig. 11. Quasi-equilibrium conditions continue until the trapped liquid is exhausted and the bubbles quickly condense from the top. Bulk liquid then moves into the heated surface, becomes superheated, and the cycle repeats.

Having already shown that only 20 per cent of the total heat flux can be attributed to bulk convection, one can argue that the remaining portion of the total heat flux is due to energy transport through the bubble. It is not possible, however, to determine this energy as was done in the pure water case. The reason for this is that in the pure water calculation we tacitly assumed that the evaporation coefficient did not vary over the bubble lifetime. This assumption cannot be justified for the reduced surface tension conditions. Obviously the evaporation coefficient will vary with time at the base of the bubble because more and more water will evaporate into the bubble leaving a larger concentration of the nonvolatile additive at the interface. Additionally, the calculation is prevented by the fact that the true value of the evaporation coefficient cannot be determined from the initial period of bubble growth (as

was done with pure water). As a bubble grows, new interface area is continuously formed, and there the concentration of additive molecules varies with the rate of diffusion of the molecules to the new surface, changing in turn the evaporation coefficient.

*Cyclic bubble formation.* The period of the gross bubble cycle varied with heat flux as shown in Figs. 8 and 10. The completely diverse trends under different conditions of surface tension can be explained by the previously hypothesized bubble growth model as follows. Under conditions of reduced surface tension, the bubble growth period is equivalent to the time required for the liquid trapped between bubbles to evaporate. This decreases as the heat flux is increased. For pure water conditions, the cycle period depends on the rate at which a thin layer of metal at the surface of the solid heater is cooled by growing bubbles. Nucleation is suppressed at the sites on the covered surface. The time required for cooling increases with heat flux.

In water containing an additive the cycle period can be calculated by determining the time required for liquid trapped at the base of the bubbles to evaporate at a given heat flux. This calculation is made in Appendix B, and the results are compared with the measured data in Table 1.

The agreement between the calculated and measured values seems quite good.

In pure water the period of the gross bubble cycle increased with increasing power settings

Table 1. Comparison of the calculated cycle period with the measured period

Heat flux (Btu/h ft <sup>2</sup> )	Calculated period (msec)	Meas. period (msec) (see Fig. 10)
$2 \times 10^5$	71	62
3	47	52
4	35	42
5	28	32
6	24	20

up to a heat flux of approximately  $6 \times 10^5$  Btu/hft<sup>2</sup>. Above this value bubbles were continuously present on the surface, and no cycling was observed.

Two possible causes of this phenomenon are: (1) pressure disturbances due to flow instabilities and (2) periodic cooling of the heater surface due to spontaneous bubble growth. A third possibility that the cycle is a result of synchronization between bubble growths and current ripple in the d.c. power source was ruled out on the basis of evidence provided by oscilloscope traces of the ripple frequencies which showed no correspondence to the bubble suppression frequencies. Additional evidence that the cycle frequency was not caused by effects due to the electrical power supply is illustrated in Table 2 where some data taken prior to this work is listed. Inspection of Table 2 shows that the suppression frequencies remain approximately the same at corresponding heat flux despite the fact that one set of data was obtained using a full wave d.c. rectifier and the other using a d.c. motor generator.

Table 2. Comparison of suppression frequencies obtained using different power sources (other conditions remaining the same)

M.G. power supply		Rectifier power supply	
Heat flux (Btu/h ft <sup>2</sup> )	Frequency (c/s)	Heat flux (Btu/h ft <sup>2</sup> )	Frequency (c/s)
$991 \times 10^3$	63.7	$956 \times 10^3$	62.1
668	79.4	632	79.4
395	134.9	378	122.3

Although precautions were taken to avoid flow instabilities, high-response pressure measurements, unfortunately, were not taken to insure that system flow effects were definitely eliminated; and hence, it is not possible to conclude that the cyclic bubble suppression was not generated by pressure disturbances. In fact, slight fluctuations were observed on the pressure gage preceding the throttling valve. The frequency of these visually appeared to increase

with heat flux which is directly opposite to the effect of increasing heat flux on the bubble cycle. This meager evidence, of course, does not preclude system flow effects as the generating source of the bubble suppression.

However, based on the measured heater surface temperature drops reported in [7] and [8], one can qualitatively argue that periodic cooling of the surface due to spontaneous bubble growth can also generate a cyclic bubble formation. This model is consistent with the theory proposed in the foregoing and is as follows.

Consider the system shown in Fig. 15 which represents a cross-section taken lengthwise

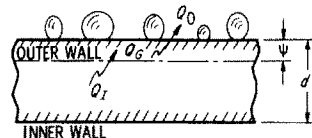


Fig. 15. Sketch of heater cross-section used to explain cyclic bubble growth in pure water.

through the heater wall. Let  $\Psi$  be the depth below the solid surface to which any thermal disturbances on the surface are felt.  $Q_I$  can be considered the internal generation in the portion of the heater which is not influenced by temperature fluctuations. For any given power setting  $Q_I$  is a constant.  $Q_0$  is the energy removed from the surface due to the boiling process. The value of  $Q_0$  is not constant and becomes very high during periods when many new bubbles are forming.  $Q_G$  is the energy generated in the thin surface layer effected by surface temperature fluctuations. An energy balance on this portion of the heater gives

$$Q_I + Q_G - Q_0(t) = \rho CV \frac{dT}{dt}. \quad (1)$$

(This is only approximate, for  $T$  has been taken as uniform through the thin surface layer treated as the system.) Now if  $Q_0(t)$  becomes greater than  $Q_I + Q_G$ , as would be the case when bubbles form spontaneously from an

unstable superheated state, one notes that  $dT/dt$  becomes negative. Hence the surface layer is cooled, and the nucleation process is suppressed. This model in essence postulates that the surface temperature fluctuation under random growing bubbles will statistically average out to a gross cyclic effect. Since the waiting period can be many times greater than the growing period in a subcooled system [4], such an assumption is not entirely unreasonable. Consider the surface during the waiting period. As  $\Delta T_{\text{sat}}$  increases certain sites will become active. Bubbles growing from these sites will cool the surface locally, while the temperature is increasing on other parts of the surface where sites requiring greater superheat are still inactive. The time required for the surface temperature to return to its original value at a cooled nucleation site is greater than that required for the site to be cooled. Thus the rate at which sites are cooled exceeds the rate at which sites are reheated (and hence generate a second bubble). As a net result the surface is cooled and becomes free of bubbles. The superheated layer is then reestablished and the cycle repeats.

This model can also explain the disappearance of the cycling effect observed at high power settings. The quantity  $(Q_I + Q_G)$  in equation (1) becomes larger with increasing power settings. Although the quantity  $Q_0$  can also increase with power settings, it obviously cannot increase indefinitely. A point is reached, therefore, where energy is generated as fast as it is removed by the spontaneous bubble formation; the surface is no longer cooled by the initial bubble growth; and the cycle stops. A satisfactory mathematical solution for determining the cycle period in pure water is not proposed here. However, an order of magnitude analysis employing equation (1) gives, for the data of Fig. 9, a predicted period of 14.8 msec which is not too greatly different from the measured 28 msec.

#### CONCLUSIONS

The present study has shown that in forced

convection nucleate boiling of water, reducing the magnitude of the liquid-vapor surface tension by the addition of a surface active agent increases both the rate of heat transfer and the number of active nucleation sites which occur at any given wall temperature. The influence on the boiling curve ( $q/A$  vs.  $\Delta T_{\text{sat}}$ ) of the parameters velocity and subcooling was insignificant over the range investigated.

High-speed motion picture studies of the ebullition process proved that the time required for the growth and collapse of bubbles in subcooled pure water is, at least, an order of magnitude smaller than that required for the growth and collapse of bubbles in subcooled water containing sufficient surface active agent to reduce the liquid-vapor surface tension by a factor of one-half. An order of magnitude analysis suggests that the different bubble growth rates are a result of different energy rate processes controlling the bubble growth. In the former case the bubble life is dependent on the rate at which energy can flow by conduction to the bubble interface. In the latter case, the presence of surfactant molecules in the bubble interface decreases the rate of evaporation to the extent that it becomes the controlling rate process.

This description of the different bubble growth mechanisms leads to a logical explanation of the cyclic formation of the gross bubble population which was experimentally observed in the present work.

An approximate analysis made to theoretically determine the magnitude of the nucleate boiling heat flux, for an individual pure water run, agrees within 16 per cent of the measured heat flux. Such agreement is probably fortuitous considering the assumptions involved in the analysis, however, it is significant to note that reasonable correspondence between calculated values and experimental values is only obtained if consideration is given not just to bulk convection due to pumping by the bubbles, as is normally the approach taken in the literature; but also to latent energy transport through the

bubbles. Both of these respective mechanisms appear to contribute significantly to the total measured heat flux.

At appreciably reduced values of surface tension the same bulk convection analysis, as made for the pure water case, shows that only 20 per cent of the heat flux is due to bubble induced convection. Hence 80 per cent, the greater part of the total heat flux, appears to result from latent heat transport. Consequently the mechanism of latent heat transport cannot be ignored if the experimental heat flux is to be explained.

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#### APPENDIX A

A sample calculation of the nucleate boiling heat flux including both energy transport through the vapor bubble and energy pumped away from the surface during the bubble growth period is carried through for the following set of measured data:

$$q/A = 299000 \text{ Btu/h ft}^2, \quad \Delta T_{\text{sat}} = 41^\circ\text{F},$$

$$T_{\text{inlet}} = 170^\circ\text{F},$$

$$\sigma = 71.4 \text{ dyn/cm (pure water),}$$

$$\text{Pres.} = 30 \text{ psia, } \text{Vel} = 0.8 \text{ ft/s, } T_w = 290^\circ\text{F}.$$

Table 3. Measurements of typical bubble sizes and growth periods

Bubble No.	Maximum height (in)	Time to grow to maximum height (msec)	Time to decay from maximum height to zero (msec)	Total time on surface (msec)
1	0.016	0.14	1.14	1.28
2	0.016	0.43	5.10	5.53
3	0.023	0.43	0.42	0.85
4	0.031	0.28	1.00	1.28
5	0.023	0.28	0.57	0.85
6	0.031	0.25	1.88	2.13
Avg.	0.023	0.30	1.67	1.97

Bubble size as a function of time is shown in Fig. 9, and the number of bubbles per unit area was evaluated from plots of which Fig. 16 is typical. A time averaged value of bubbles per unit area was obtained by graphical integration. The average for two typical curves was 1570 bubbles/sq. in.

A bubble is assumed to grow according to the model shown in Fig. 12, and according to Fig. 9 grows to approximately 0.020 in. in 0.1 msec. During this growth period it is assumed that the bubble is completely surrounded by super-



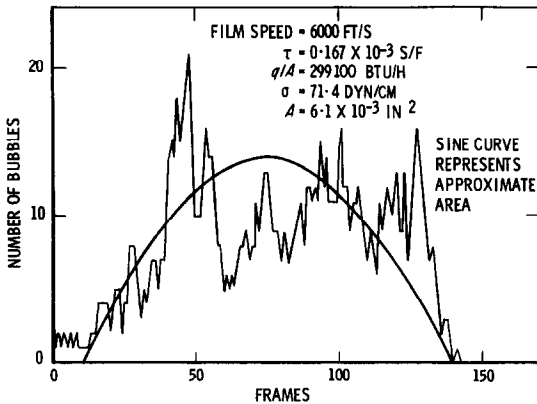


FIG. 16. Bubble on area,  $A$ , per individual picture frame.

heated liquid and that energy is transferred to the bubble over its entire surface.

From kinetic theory the evaporation rate at an interface is given by

$$\frac{dM}{A dt} = \alpha m \frac{n\bar{v}}{4},$$

where

$$\bar{v} = \left( \sqrt{\frac{8KT}{\pi m}} \right), \tag{2}$$

and  $\alpha$  is the evaporation coefficient defined as the fraction of molecules which escape from the liquid surface. During the initial growth of the bubble  $\alpha$  can be evaluated by equating  $\alpha h_{fg} m n \bar{v} / 4$  to the heat flux because it is postulated that only during this time does evaporation occur over the entire bubble surface. For this purpose we can calculate the heat flux for bubble growth from

$$q/A = \rho_v h_{fg} V_b / \bar{A}_b \tau. \tag{3}$$

The time averaged bubble area  $\bar{A}_b$  is determined assuming linear initial bubble growth with time (see Fig. 9). The ratio  $V_b / \bar{A}_b$  then becomes  $D_b / 2$  regardless of whether a spherical shape or a hemispherically capped cylinder suggested by the high-speed photographs is used. The calculated heat flux for initial bubble growth becomes 570 Btu/sft<sup>2</sup> and for  $\bar{v}$  evaluated at

$T_w$  the final result for  $\alpha$  is 0.015. Literature values range from 1.0 to 0.034 for clean water at room temperature hence this seems reasonable, considering that traces of impurities can substantially alter these values.

Before proceeding with the calculation of latent energy transport it is necessary to evaluate the temperature drop of the heated surface due to the rapid energy removal occurring locally beneath the growing bubble. A numerical solution of the one-dimensional heat-conduction problem, treating the heating surface as an infinite slab with internal generation, was obtained by a computer. This showed that for conditions involving the high heat-transfer rates (570 Btu/sft<sup>2</sup>) and short time periods (0.1 msec) internal generation can be neglected and the temperatures calculated as for a semi-infinite solid with constant flux from the surface. Thus the temperature drop on the surface under a growing bubble is given approximately by the well-known relation (Carslaw and Jaeger [11])

$$T - T_i = \frac{2q}{KA} \left( \frac{\lambda t}{\pi} \right)^{\frac{1}{2}} \tag{4}$$

and the surface temperature drop in 0.1 msec becomes 15 degF, therefore  $T'_w = 275^\circ\text{F}$  (after the growth period).

Returning to the consideration of the bubble growth, it is postulated that the top of the bubble after the first 0.1 msec of the growth period breaks through the thermal layer and contacts fluid at  $T_B = 170^\circ\text{F}$ . When this occurs the top of the bubble begins to condense while the base is still evaporating. Mass is then transferred through the bubble. At first the evaporation at the base will exceed the condensation at the top and the bubble will grow. This can be seen from Fig. 9 to last, on an average, approximately 2 msec. However, as energy is being removed from the region surrounding the base at a rate which is greater than it can be generated or flow by conduction, the bubble base becomes cooled. Thus the condensation rate exceeds the evaporation rate and the bubble

starts to collapse. To evaluate the heat flux through the bubble under these conditions the following approximation is made. Assume a stream of molecules given by  $\alpha n \bar{v}(T_w)/4$  is coming from the base of the bubble and a stream given by  $\alpha n \bar{v}(T_b)/4$  from the top. Then the net mass transfer through the cross-sectional area of the bubble is approximately given by the difference in these two streams, times the mass of a molecule

$$\frac{dM}{dt} = \frac{\alpha mn}{4} [\bar{v}(T_w) - \bar{v}(T_b)] \quad (5)$$

and the approximate heat flux is

$$q/A = \frac{\alpha h_{fg} mn}{4} [\bar{v}(T_w) - \bar{v}(T_b)]. \quad (6)$$

Assuming that evaporation takes place as long as  $T_w > T_{\text{sat}}$ ,  $\bar{v}(T_w)$  is evaluated using an average wall temperature  $(T'_w + T_{\text{sat}})/2$ . Where  $T'_w$  is the calculated wall temperature after the initial 0.1 msec of bubble growth (275°F in this case). Solving this for  $q/A$  gives a value of 38.6 Btu/sft<sup>2</sup>.

Based on this heat flux existing over a period of time equal to the average bubble life period minus the initial 0.1 msec, the final wall temperature is computed from equation (4) to be 267°F which is approximately saturation temperature (250°F).

The heat flux due to mass transport through the bubble can now be determined by weighting the heat flux according to time (during growth-collapse cycle). This yields  $q/A = 65.5$  Btu/sft<sup>2</sup>, which when reduced by the factor (70 per cent) of surface covered with bubbles reduces to  $165 \times 10^3$  Btu/hft<sup>2</sup>, or 55 per cent of the measured ( $229 \times 10^3$ ) flux.

In addition to transport through the bubble, there is also energy transfer by bulk convection produced when high temperature liquid is pumped away from the heated surface by a growing vapor bubble. This has been discussed in the introduction. A modification of the approach used by Han and Griffith cited

earlier to take into account the initial temperature distribution and the surface cooling effect follows.

Instead of modeling the liquid layer as a semi-infinite solid as used in [5] for saturated pool boiling, we treat the liquid contacting the heated surface as a slab (of thickness  $\delta$ ) as suggested by Hsu and Graham [4]. Since the bubbles do not detach, but collapse back on to the heated surface, the slab of liquid strikes the solid heater surface with some initial temperature distribution. Finally, the assumption of a constant wall temperature is not valid if consideration is given to the surface cooling as discussed previously. However, consideration of a varying wall temperature makes the solution of the conduction equation for an infinite slab rather unwieldy. Therefore in the following analysis the wall temperature will be assumed constant, but when evaluating the resulting expression numerically the average wall temperature will be used. The temperature distribution in the slab, when it initially contacts the wall, is obtained assuming that at the beginning of the bubble collapse the top of the bubble is in contact with a slab of fluid at a uniform temperature  $T_b$ . Let  $\delta$  be the distance from the heated surface beyond which the liquid temperature remains essentially uniform at  $T_b$ . The surface  $x = 0$  is assumed to be suddenly changed to and maintained at a temperature  $T_{\text{sat}}$ . Then the transient one-dimensional conduction equation is solved subject to the boundary conditions as follows:

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\lambda} \frac{\partial \theta}{\partial t},$$

$$\theta = T - T_b = 0, \quad x = \delta;$$

$$\theta = \theta_s, \quad x = 0; \quad t = 0. \quad (7)$$

The solution is:

$$\theta = \theta_s(1 - \eta) + \frac{2}{\pi} \sum_0^{\infty} -\frac{\theta_s}{n} \times \sin(n\pi\eta) \exp(-\pi^2 n^2 \tau);$$

where

$$\eta = \frac{x}{\delta} \quad \tau = \frac{\lambda t}{\delta^2} \quad (8)$$

The initial temperature distribution is given by this expression when  $t$  is equal to the collapse time of the bubble.

Next the slab can be considered as being in contact with the heated wall. The temperature distribution at the time when the next bubble grows is then found from the solution of the one-dimensional conduction equation subject to boundary condition.

$$\begin{aligned} \theta &= \theta(x) \text{ [given by equation (8)],} & t &= 0; \\ \theta &= \theta_w, & x &= 0; \quad \theta = 0, & x &= \delta. \end{aligned}$$

This solution [11], after evaluating the integral involved in the general solution, is

$$\begin{aligned} \theta &= \theta_w(1 - \eta) + (2/\pi) \sum_1^{\infty} \theta_s [1 - C'(n)] \\ &\quad - \theta_w(1/n) \sin(n\pi\eta) \exp(-n^2\pi^2\tau) \end{aligned} \quad (9)$$

where

$$C'(n) = \exp\left(\frac{-n^2\pi^2\lambda t_w}{\delta^2}\right).$$

Substitution of  $t_w$  (the waiting period) into equation (9) gives the temperature distribution in the liquid when the bubble grows.

Now following Han and Griffith [5]

$$Q = \int_0^{\delta} \theta(t_w) C_L \rho_L dx$$

Substituting equation (9) into the integral and integrating gives

$$\begin{aligned} Q &= \rho_L C_L \theta_w \delta \left\{ \frac{1}{2} - \frac{4}{\pi^2} \left[ \sum_1^{\infty} \left( 1 - \frac{\theta_s}{\theta_w} \right) \right. \right. \\ &\quad \left. \left. \times [1 - C'(n)] \frac{C''(n)}{n^2} \right] \right\} \quad (10) \\ &\quad n = 1, 3, 5 \dots \end{aligned}$$

where

$$C''(n) = \exp\left(\frac{-n^2\pi^2\eta t_w}{\delta^2}\right).$$

To evaluate equation 10  $\delta$  and  $t_w$  must be determined. Hsu [12] gives the following expression for  $\delta$ .

$$\frac{\delta(\theta_w - \theta_s)^2}{\theta_w} \leq \frac{12.8 \sigma T_{sat}}{h_f g \rho_v} \quad (11)$$

Although Hsu did not include an initial temperature distribution in his analysis the value of  $\delta$  will not be appreciably altered. The result is  $\delta = 4.1 \times 10^{-4}$  ft. Solving for  $t_w$  by the method of Hsu gives a value smaller than the bubble growth time, which from the experimental results of this work is known to be correct. However, Hsu did not consider a varying wall temperature. If the approximate time for the surface to recover is included in the waiting period the value of  $t_w$  is 10.2 msec. Using these values for  $\delta$  and  $t_w$  in equation 10 gives  $Q = 1.0$  Btu/ft<sup>2</sup> bubble. The heat flux due to pumping is then given by

$$q/A = Q \frac{\pi D_b^2 N}{4 A \tau} \quad (12)$$

where  $N/A$  is the number of new bubbles or growing bubbles. This value is obtained by multiplying the time averaged value of bubbles per unit area (1570 bubbles/in<sup>2</sup>) by the ratio of the time for a bubble to grow to its maximum size (0.3 msec) to its total lifetime (1.97 msec). Han and Griffith chose the area influenced by a growing bubble as  $\pi(2D_b)^2/4$ . However, only the actual bubble base area is used here, because the former expression gives an area greater than the total heater surface area. Solving for  $q/A$  gives 184000 Btu/ft<sup>2</sup>h. Combining this value with the heat flux due to energy transport through the bubble, the total heat flux is obtained.

$$(q/A)_T = (q/A)_L + (q/A)_C = 349000 \text{ Btu/ft}^2 \text{ h.} \quad (13)$$

This is 16 per cent higher than the measured value.

## APPENDIX B

The following is an analysis of the cyclic bubble growth phenomenon in water with reduced surface tension. It is postulated that the cycle is caused by the bubble formation trapping liquid between bubbles during the initial growth. This trapped liquid then sustains the bubbles by supplying liquid for evaporation at the base of the bubbles. When the liquid is completely evaporated the bubbles collapse. The bubble configuration is treated as a hexagonal close pack. Liquid is assumed to be trapped between the bubbles to a depth  $D_b/2$ , where  $D_b$  is the diameter at which the bubbles first contact each other. This value measured on high speed pictures is 0.017 in and remains approximately the same for all heat fluxes. The volume of liquid trapped per unit area for the described geometry is:

$$\frac{[(\sqrt{\frac{3}{8}}) - (\pi/16)^3] D_b}{(\sqrt{\frac{3}{4}}) D_b^2} = 0.05 D_b. \quad (14)$$

This bubble formation will persist on the surface for a period as long as is required to evaporate a volume of liquid equal to  $0.05 D_b$ . Hence

$$\tau = \frac{0.05 D_b \rho_L h_{fg}}{(q/A)}. \quad (15)$$

The results of the calculation are given in Table 1. (See "Discussion of Results").

A very approximate value for the evaporation coefficient  $\alpha$  can be determined by the following expression

$$\alpha = \frac{0.05 D_b \rho_L A dt}{dM} = 2.5 \times 10^{-4}.$$

It is apparent that the value of  $\alpha$  is considerably reduced from that of pure water. This results from the molecules of the additive being present at the interface. Note that  $\alpha$  is an average value which varies as the water evaporates leaving a higher concentration of additive at the interface.

Based on this value the approximate heat flux at the surface is

$$q/A = \alpha h_{fg} \frac{dM}{A dt} = 9.5 \text{ Btu/sft}^2.$$

Calculating the surface temperature drop due to this heat flux for  $\tau_{\text{avg}} = 42$  msec gives, from equation (4),  $\Delta T = 4.3$  degF.

**Résumé**— On décrit une étude expérimentale de l'ébullition par convection forcée dans un tuyau annulaire vertical. Le fluide en ébullition était de l'eau soit pure soit avec différentes quantités d'agents tensio-actifs destinés à réduire la tension superficielle.

En dehors de celle-ci, on a fait varier également les paramètres de vitesse et de sous-refroidissement.

Pour la même température pariétale, on a mesuré un flux plus élevé et une population de bulles plus grande pendant l'ébullition de l'eau avec tension superficielle réduite. En outre, la vitesse de croissance des bulles était plus faible que dans l'eau pure, d'un facteur au moins de dix, et les bulles se formaient toutes en même temps plutôt que dans des sites isolés.

La différence entre ce type de croissance de bulles et celui dans l'eau pure a été attribuée à des processus différents de transport d'énergie contrôlant la croissance des bulles. On a supposé que la vitesse de conduction de la chaleur à la surface contrôle la croissance des bulles dans l'eau pure et la vitesse d'évaporation dans l'eau contenant un additif. Un modèle dans lequel on introduit cette hypothèse est proposé et l'on présente un calcul d'ordre de grandeur afin de vérifier le modèle.

On en conclut que le transport d'énergie latente contribue à environ 50% du flux de chaleur mesuré dans l'eau sans additifs tensioactifs et sensiblement plus dans de l'eau contenant un additif.

**Zusammenfassung**—Die Arbeit behandelt eine experimentelle Untersuchung des Siedens bei Zwangskonvektion in einem vertikalen Ringspalt. Die siedende Flüssigkeit bestand sowohl aus reinem Wasser als auch aus Wasser, das mit verschiedenen Mengen von Oberflächenaktivierungstoffen versetzt war, um die Oberflächenspannung herabzusetzen. Als Parameter wurden zusätzlich zur Oberflächenspannung auch die Geschwindigkeit und die Unterkühlung variiert.

Bei derselben Wandtemperatur wurde beim Sieden von Wasser mit reduzierter Oberflächenspannung

eine höhere Wärmestromdichte und eine grössere Blasenbesetzung gemessen. Die Phasenwachstumsgeschwindigkeit war wenigstens um einen Faktor 10 geringer als in reinem Wasser und die Blasen bildeten sich eher zusammenhängend als an einzelnen Keimstellen.

Der Unterschied bei dieser Art des Blasenwachstums im Gegensatz zu der von reinem Wasser wird verschiedenen das Blasenwachstum bestimmenden Arten des Energietransportes zugeschrieben. Es wird angenommen, dass das Blasenwachstum in reinem Wasser vom Betrag der Wärmeleitung in der Oberfläche und in Wasser mit einem Zusatz von der Verdampfungsrate bestimmt wird. Es wird ein auf dieser Hypothese beruhendes Modell vorgeschlagen und zur Betätigung dieses Modells eine Analyse der Grössenordnungen angegeben.

Man kann folgern, dass der Transport latenter Energie in Wasser ohne Oberflächenaktivierungszusätze etwa 50% der gemessenen Wärmestromdichte und in Wasser mit einem Additiv beträchtlich mehr ausmacht.

**Аннотация**—Приводится экспериментальное исследование кипения при вынужденной конвекции в вертикальном канале. В качестве рабочей жидкости использовалась вода как чистая, так и с примесью различных реагентов, уменьшающих поверхностное натяжение. Кроме поверхностного натяжения изменялись скорость и тепловой поток.

При одной и той же температуре стенки наибольший тепловой поток и плотность образования пузырьков наблюдались при кипении воды, содержащей поверхностно-активные добавки. Также отмечается, что интенсивность роста пузырьков была по меньшей мере на порядок меньше, чем в чистой воде, и что пузырьки образовывались по всей поверхности, а не на отдельных участках.

Отличие процессов образования пузырьков при кипении воды, содержащей упомянутые вещества, от процессов, проходящих в чистой воде, вызывается различным механизмом энергопереноса, определяющим рост пузырьков. Выдвигается гипотеза о том, что интенсивность теплопереноса на поверхности влияет на рост пузырьков в чистой воде и на интенсивность испарения воды, содержащей добавки. Предложена модель, демонстрирующая эту гипотезу, и приводится количественный анализ для проверки модели.

Делается вывод о том, что скрытая энергия переноса составляет около 50% измеренного теплового потока в воде без поверхностно-активных добавок и гораздо большую долю в воде, содержащей такие добавки.